Minutes of BOS in Mathematics (PG)
Held on 24.09.2016 at In Dept. of Mathematics
Osmania University.
The following members were present on resolved.

1. prof. M.V. Ramana Murthy, Chairman [signature]
2. prof. B. Shankar, Member [signature] 24/9/16
3. prof. M. Rangamma Member M. Ram[signature] 24-9-2016
4. Dr. M. Chenna Krishna Reddy Member
5. Dr. N. Kishan Member [signature]
6. Dr. Alka Mashalker Member Alka Mashal[signature] 24/9/16
7. Dr. C. Govardhan, Member [signature] 24.9.16
8. Mr. M. Madhu, Member [signature] 24/9/16

The following resolutions were made

1. It is resolved to approve the proposed Syllabus for P.G Course offered by University College and affiliated Colleges of ... Under CBCS base

It is resolved to follow the approved Syllabus from the academic years 2016-17 and 2017-18.

3. It is resolved to follow the approved Syllabus for P.G Course of Palamuru University in Telangana State.

4) It is resolved to follow the same evaluation scheme as well as panel to examiners ~~for~~ suggested by the BOS in Maths (PG) of Osmania University, Hyderabad.

24/9/16

M. Ram
24-9-2016

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Departemnt of MATHEMATICS,PU
Proposed Choice Based Credit System (CBCS)
M.Sc Mathematics

Semester -I

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 101	I	Algebra	4	20	80	100	4
2. Core	MM 102	II	Analysis	4	20	80	100	4
3. Core	MM 103	III	Mathematical Methods	4	20	80	100	4
4. Core	MM 104	IV	Elementary Number Theory	4	20	80	100	4
5. Practical	MM 151	Practical	Algebra	4	....	50	50	2
6. Practical	MM 152	Practical	Analysis	4	....	50	50	2
7. Practical	MM 153	Practical	Mathematical Methods	4	....	50	50	2
8. Practical	MM 154	Practical	Elementary Number Theory	4	....	50	50	2
			Total :	32				24

M. Ram 24/9/16 24-9-2016 Afl Mashal 24/9/16 24/9/16 24.9.16

DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. Mathematics

Algebra

MM 101 Paper I Semester I

Unit I

Automaphisms- Conjugacy and G-sets- Normal series solvable groups- Nilpotent groups. (Pages 104 to 128 of [1])

Unit II

Structure theorems of groups: Direct product- Finitly generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders p^2,pq . (Pages 138 to 155)

Unit III

Ideals and homomsphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma (Pages 179 to 211).

Unit-IV

Unique factorization domains - Principal ideal domains- Euclidean domains- Polynomial rings over UFD- Rings of traction.(Pages 212 to 228)

Text Books:

[1] Basic Abstract Algebra by P.B. Bhattacharya, S.K. Jain and S.R. Nagpanl.

Reference: 1] Topics in Algebra by I.N. Herstein.

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M.Sc (Mathematics)

Algebra

MM 151 **Paper I** **Semester I**

Practical Questions

1. A finite group G having more than two elements and with the condition that $x^2 \neq e$ for some $x \in G$ must have nontrivialautomorphism.
2. (i) Let G be a group Define $a * x = ax,\ a, x \in G$ then the set G is a G-set

   (ii) Let G be a group Define $a * x = axa^{-1}\ a, x \in G$ then G is a G-set.
3. An abelian group G has a composition series if and only if G is finite
4. Find the number of different necklaces with p beads p prime where the beads can have any of n different colours
5. If G is a finite cyclic group of order n then the order of Aut G, the group of automorphisms of G, is $\phi(n)$, where ϕ is Euler's function.
6. If each element $\neq e$ of a finite group G is of order2 then $|G| = 2^n$ and $G \approx C_1 \times C_2 \times \ldots\ldots \times C_n$ where C_i are cyclic and $|C_i| = 2$.
7. (i) Show that the group $\frac{Z}{<10>}$ is a direct sum of $H = \{\bar{0}\ \bar{5}\}$ and $K = \{\bar{0}\ \bar{2}\ \bar{4}\ \bar{6}\ \bar{8}\}$

   (ii) Show that the group $\left(\frac{z}{\langle 4 \rangle}, +\right)$ cannot be written as the direct sum of two Subgroups of order 2.
8. (i) Find the non isomorphic abelian groups of order 360

   (ii) If a group of order p^n contains exactly one sub group each of orders $p, p^2, \_\ \_\ \_ P^{n-1}$ then it is cyclic.
9. Prove that there are no simple groups of orders 63, 56, and 36
10. Let G be a group of order 108. Show that there exists a normal subgroup of order 27 or 9.
11. (i) Let **R** be acommutative Ring wilth unity. Suppose R has no nontrivial ideals .Prove that R is a field.

    (ii) Find all ideals in Z and in $\frac{Z}{\langle 10 \rangle}$

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12. (i) The only Homomorphism from the ring of integers Z to Z are the identity and Zero Mappings.

    (ii) Show that any nonzero homomorphism of a field F into a ring R is one-one.

13. For any tow ideals A and B in a Ring R (i) $\frac{A+B}{B} \approx \frac{A}{A \cap B}$

    (ii) $\frac{A+B}{A \cap B} \approx \frac{A+B}{A} \times \frac{A+B}{B} \approx \frac{B}{A \cap B} \times \frac{A}{A \cap B}$ In particular if $R = A+B$ then $\frac{R}{A \cap B} \approx \frac{R}{A} \times \frac{R}{B}$.

14. Let R be a commutative ring with unity in which each ideal is prime then R is a field

15. Let R be a Boolean ring then each prime ideal $P \neq R$ is maximal.

16. The commutative integral domain $R = \{a + b\sqrt{-5} / a, b \in Z\}$ is not a UFD.

17. (i) The ring of integers Z is a Euclidean domain

    (ii) The Ring of Gausion Integers $R = \{m + n\sqrt{-1} / m, n \in Z\}$ is a Euclidean domain

18. (i) Prove that $2+\sqrt{-5}$ is irreducible but not prime in $Z(\sqrt{-5})$

    (ii) Show that $1+2\sqrt{-5}$ *and* 3 are relatively prime in $Z(\sqrt{-5})$

19. Let R be a Euclidean domain . Prove the following

    (i) If $b \neq 0$ *then* $\phi(a) < \phi(b)$

    (ii) If a and b are associates then $\phi(a) = \phi(b)$

    (iii) If a/b *and* $\phi(a) = \phi(b)$ *then* a and b are associates

20. Prove that every nonzero prime ideal in a Euclidean domain is maximal.

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DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. Mathematics

Semester I

MM – 102

Analysis

Paper-II

Unit I

Metric spaces- Compact sets- Perfect sets- Connected sets

Unit II

Limits of functions- Continuous functions- Continuity and compactness Continuity and connectedness- Discontinuities – Monotone functions.

Unit III

Rieman- Steiltjes integral- Definition and Existence of the Integral- Properties of the integral- Integration of vector valued functions- Rectifiable waves.

Unit-IV

Sequences and series of functions: Uniform convergence- Uniform convergence and continuity- Uniform convergence and integration- Uniform convergence and differentiation- Approximation of a continuous function by a sequence of polynomials.

Text Books:

[1] Principles of Mathematical Analysis (3rd Edition) (Chapters 2, 4, 6) By Walter Rudin, Mc Graw-Hill Internation Edition.

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M.Sc. Mathematics

Analysis

MM 152 Paper –II Semester –

IPractical Questions

1. Construct a bounded set of real numbers with exactly three limit points
2. Suppose E^1 is the set of all limit points of E. Prove that E^1 is closed also prove that E and E have the same limit points.
3. Let E^0 demote the set of all interior points of a set E. Prove that E^0 is the largest open set contained in E Also prove that E is open if and only if $E = E^0$
4. Let Xbe an infinite set.For $p \in X, q \in X$ define

$$d(p,q) = \begin{cases} 1 \ \text{if} \ p \neq q \\ 0 \ \text{if} \ p = q \end{cases}$$

Prove that this is a metric, which subsets of the resulting metric space are open, which areclosed? Which are compact?

5. i) If A and B are disjoint closed sets in some metric space X, prove that they are separated
ii) Prove the same for disjoint open sets
iii)Fixa $p \in X$ and $\delta > o$, Let A = $\{ \ q \in X : d(p,q) < \delta \}$
and $B = \{q \in X \, {:}d(p,q) > \delta \}$ prove that A and B are separated.

6. i) Suppose f is a real function on R which satisfies $\lim_{h \to o} [f(x+h) - f(x-h) = o]$ for every $x \in R$ Does this imply that f is continuous? Explain

ii) Let f be a continuous real function on a metric space X,let Z(f) = $\{p \in X : f(p) = 0\}$ prove that z (f) is closed.

7. If f is a continuous mapping of a metric space X into a metric space Y .prove that

$$f(\overline{E}) \subset \overline{f(E)} \quad \text{for every set } E \subset X$$

8. Let f and g be continuous mapping of a metric space X into a metric space Y Let E be a dense subset of X. Prove that
   i) f(E) is dense in f(X)
   ii) If g(p) =f(p) $\forall$ p $\in$ E , Prove that g(p) =f(p) $\forall$ p $\in$X

9. Suppose f is a uniformly continuous mapping of a metric space X into a metric space Y and $\{X_m\}$ is a Couchy sequence in X prove that $\{f(X_m)\}$ is Cauchy sequence in Y

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10. Let I = [0 , 1] be the closed unit interval, suppose f is a continuous mapping of f into I. Prove that f(x) = x for at least one x

11. Suppose α increases on [a , b] , a <x_0<b, α is continuous at x_0, f(x_0) = 1 and f(x) =0 if x $\neq x_0$

.Prove that f $\in$R(α) and $\int_a^b f\,d\alpha = 0$

12. Suppose f$\geq$0 and f is continuous on [a , b] and $\int_a^b f(x)dx = 0$, Prove that f(x) =0 $\forall$ x ∈ [a , b]

13. If f(x) = 1or 0 according as x is rational or not .Prove that f $\notin R$ on [a , b] for any a,b,∈R with a<b.Also prove that f $\notin$R(α) on [a , b] with respect to any monotonically increasing function α on [a , b]

14. Suppose f is a bounded real function on [a , b] and f^2∈R on [a , b]. Does it follow that $f \in R$?

Does the answer change if we assume that f^3∈R?

15. Suppose γ_1 *and* γ_2 are the curves in the complex plane defined on [0,2 π] by $\gamma_1(t) = e^{it}$, $\gamma_2(t) = e^{2it}$

Show that the two curves have the same range

Also Show that γ_1 *and*γ_2 are rectifiable and find the curve length of γ_1 *and*γ_2

16. Discuss the uniform conversance of the sequence of functions {f_n} where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}} \quad \text{x real, n = 1,2,3....}$$

17. Give an example of a series of continuous functions whose sum function may be discontinuous.

18. Discuss the uniform conversance of the sequence

$$f_n(x)=\frac{1}{1+nx} x > 0,\ n = 1,2,3...$$

19. Give an example of a sequence of functions such that

$$\lim \int f_n \neq \int \lim f_n$$

20. Prove that a sequence {f_n} converse to f with respect to the metric of C(x) if and only if $f_n \rightarrow f$ uniformly on X

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DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. (Mathematics)

MM – 103 Semester I

Mathematical Methods

Paper- III

Unit I

Existence and Uniqueness of solution of $\frac{dy}{dx} = f(x,y)$. The method of successive approximation- Picard's theorem- Sturm-Liouville's boundary value problem.
Partial Differential Equations: Origins of first-order PDES-Linear equation of first-order-Lagrange's method of solving PDE of $P_p+Qq = R$ – Non-Linear PDE of order one-Charpit method- Linear PDES with constant coefficients.

Unit II

Partial Differential Equations of order two with variable coefficients- Canonical form Classification of second order PDE- separation of variable method solving the one-dimensional Heat equation and Wave equation- Laplace equation.

Unit III

Power Series solution of O.D.E. – Ordinary and Singular points- Series solution about an ordinary point -Series solution about Singular point-Frobenius Method.
Lagendre Polynomials: Lengendre's equation and its solution- Lengendre Polynomial and its properties- Generating function-Orthogonal properties- Recurrance relations- Laplace's definite integrals for P_n (x)- Rodrigue's formula.

Unit-IV

Bessels Functions: Bessel's equation and its solution- Bessel function of the first kind and its properties- Recurrence Relations- Generating function- Orthogonality properties.
Hermite Polynomials: Hermite's equation and its solution- Hermite polynomial and its properties- Generating function- Alternative expressions (Rodrigue's formula)- Orthogonality properties- Recurrence Relations.

Text Books:

[1] "Elements of Partial Differential Equations", By Ian Sneddon, Mc.Graw-Hill International Edition.
[2] "Text book of Ordinary Differential Equation", By S.G.Deo, V. Lakshmi Kantham, V. Raghavendra, Tata Mc.Graw Hill Pub. Company Ltd.
[3] "Ordinary and Partial Differential Equations", By M.D. Raisingania, S. Chand Company Ltd., New Delhi.

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M.Sc. Mathematics
Mathematical Methods

MM 153 — Paper III — Semester I

Practical Questions

1. Compute the first three successive approximations for the solution of the initialvalue problem $\frac{dx}{dt} = x^2, \quad x(0) = 1$.

2. Solve $yp = 2yx + \log q$.

3. Solve $yzp + zxq = xy$ with usual notations.

4. Explain Strum-Liouille's boundary value problems.

5. Classify the equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0.$

6. Solve $r + t + 2s = 0$ with the usual notations.

7. Find the particular integral of the equation $(D^2 - D)Z = e^{2x+y}$.

8. Solve in series the equation $xy'' + y' - y = 0$.

9. Solve $y'' - y = x$ using power series method.

10. Solve the Froenius method $x^2y'' + 2x^2y' - 2y = 0$.

11. Solve in series $2xy'' + 6y' + y = 0$.

12. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is an integer.

13. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

14. Prove that $H_n(-x) = (-1)^n H_n(x)$.

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15. Show that $H_{2n+1}(0) = 0$.

16. Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.

17. Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$; *with* $u(x,0) = 4e^{-x}$ using separation of variable method.

18. Find the surface passing through the parabolas $Z = 0,\ y^2 = 4ax$ *and* $Z = 1,\ y^2 = -4ax$ and satisfying the equation $xr + zp = 0$.

19. Find the surface satisfying $t = 6x^2y$ containing two lines $y = 0 = z$ and $y = 2 = z$.

20. Reduse the equation $x^2r - y^2t + px - qy = x^2$ in the canonical form.

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M.Sc. Mathematics
Semester I

MM104 **Elementary Number Theory**

Paper- IV

UNIT-I

The Division Algorithm- Number Patterns- Prime and Composite Numbers- Fibonacci and Lucas' numbers- Fermat Numbers- GCD-The Euclidean Algorithm- The Fundamental Theorem of Arithmetic- LCM- Linear Diophantine Equations

UNIT-II

Congruences- Linear Congruences- The Pollard Rho Factoring Method- Divisibility Tests- Modular Designs- Check Digits- The Chinese Remainder Theorem- General Linear Systems- 2X2 Systems

UNIT-III

Wilson's Theorem- Fermat's Little Theorem- Pseudo primes- Euler's Theorem- Euler's Phi function Revisisted- The Tau and Sigma Functions- Perfect Numbers- Mersenne Primes- The Mobius Function

UNIT-IV

The Order of a Positive Integer- Primality Tests- Primitive Roots for Primes- Composites with Primitive roots- The Algebra of Indices- Quadratic Residues- The Legendre Symbol- Quadratic Reciprocity- The Jacobi Symbol

Text Book : Thomas Koshy , *Elementary Number Theory with Applications*

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Elementary Number theory

Practical Questions

MM154 Paper IV Semester I

1

Find the positive integer a if $[a, a+1] = 132$.

2

Find the twin primes p and q such that $[p, q] = 323$.

3

The LDE $ax + by = c$ is solvable if and only if $d|c$, where $d = (a, b)$. If x_0, y_0 is a particular solution of the LDE, then all its solutions are given by

$$x = x_0 + \left(\frac{b}{d}\right)t \quad \text{and} \quad y = y_0 - \left(\frac{a}{d}\right)t$$

where t is an arbitrary integer.

4

Solve the LDE $1076x + 2076y = 3076$ by Euler's method.

5

Find the general solution of each LDE

$2x + 3y = 4$

$12x + 13y = 14$

6

Determine the number of incongruent solutions of each linear congruence.

$$12x \equiv 18 \pmod{15}$$
$$28u \equiv 119 \pmod{91}$$
$$49x \equiv 94 \pmod{36}$$

7

Using congruences, solve each LDE.

$3x + 4y = 5$

$15x + 21y = 39$

8

Using the Pollard rho method, factor the integer 3893.

9

Prove that the digital root of the product of twin primes, other than 3 and 5, is 8.

10

Using the CRT, solve Sun-Tsu's puzzle:

$$x \equiv 1 \pmod 3), \quad x \equiv 2 \pmod 5), \quad \text{and} \quad x \equiv 3 \pmod 7)$$

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11

Prove each, where p is a prime.

Let p be odd. Then $2(p-3)! \equiv -1 \pmod{p}$.

$(p-1)(p-2)\cdots(p-k) \equiv (-1)^k k! \pmod{p}$, where $1 \le k < p$.

12

Find the remainder when 24^{1947} is divided by 17.

13

Let p be any odd prime and a any nonnegative integer. Prove the following.

$1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 \pmod{p}$

$1^p + 2^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$

14

Verify each.

$(12+15)^{17} \equiv 12^{17} + 15^{17} \pmod{17}$

$(16+21)^{23} \equiv 16^{23} + 21^{23} \pmod{23}$

15

Find the remainder when 245^{1040} is divided by 18.

16

Evaluate $(-4/41)$ and $(-9/83)$.

17

Verify that $9973 | (2^{4986}+1)$.

18

Prove that there are infinitely many primes of the form $4n+1$.

19

Show that $1!+2!+3!+\cdots+n!$ is never a square, where $n > 3$.

20

Prove that there are infinitely many primes of the form $10k-1$.

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Proposed Choice Based Credit System (CBCS)
M.Sc Mathematics

Semester -II

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 201	I	Advnaced Algebra	4	20	80	100	4
2. Core	MM 202	II	Advnaced Analysis	4	20	80	100	4
3. Core	MM 203	III	Theory of Ordinary differential equation	4	20	80	100	4
4. Core	MM 204	IV	Topology	4	20	80	100	4
5. Practical	MM 251	Practical	Advanced Algebra	4	....	50	50	2
6. Practical	MM 252	Practical	Advnaced Analysis	4	....	50	50	2
7. Practical	MM 253	Practical	Theory of Ordinary differential equation	4	....	50	50	2
8. Practical	MM 254	Practical	Topology	4	....	50	50	2
			Total :	32				24

24/9/16. M. Ram 24-9-2016 24/9/16 24/9/16 24.9.16

DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. (Mathematics)

MM –201 Semester II

Advanced Algebra

Paper I

Unit I

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion- Adjunction of roots- Algebraic extensions-Algebraically closed fields (Pages 281 to 299)

Unit II

Normal and separable extensions: Splitting fields- Normal extensions- Multiple roots- Finite fields- Separable extensions (Pages 300 to 321)

Unit III

Galois theory: Automorphism groups and fixed fields- Fundamental theorem of Galois theory- Fundamental theorem of Algebra (Pages 322 to 339)

Unit-IV

Applications of Galoes theory to classical problems: Roots of unity and cyclotomic polynomials- Cyclic extensions- Polynomials solvable by radicals- Ruler and Compass constructions. (Pages 340-364)

Text Books:

[1] Basic Abstract Algebra- S.K. Jain, P.B. Bhattacharya, S.R. Nagpaul.

Reference Book: Topics in AlgrbraByI. N. Herstein

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M.Sc. Mathematics

Advanced Algebra

MM 251 **Paper I** **Semester II**

Practical Questions

1. (i) $\phi_p(x) = 1 + x + ... + x^{p-1}$ is irreducible over Q. Where p is a prime.

   (ii) Show that $x^3 + 3x + 2 \in \frac{Z}{\langle 7 \rangle}(x)$ is irreducible over the field $\frac{Z}{\langle 7 \rangle}$.

2. Show that the following polynomials are irreducible over Q

   (i) $x^3 - x - 1$ (ii) $x^4 - 3x^2 + 9$ (iii) $x^4 + 8$

3. Show that there exists an extension of E *of* $\frac{Z}{\langle 3 \rangle}$ with nine elements having all the roots of $x^2 - x - 1 \in \frac{Z}{\langle 3 \rangle}(x)$

4. (i) Show that there is an extension E of R having all the roots of $1 + x^2$

   (ii) Let $f_i(x) \in F(x)$ for i= 1, 2, ...m then there exists an extension E of F in which each polynomial has root

5. Show that $\sqrt{2}$ *and* $\sqrt{3}$ are algebraic over Q and find the degree of $Q(\sqrt{2})$ over Q and $Q(\sqrt{3})$ over Q.

   (iii) Find a suitable number a such that $Q(\sqrt{2}, \sqrt{5}) = Q(a)$.

6. Show that the degree of the extension of the splitting field of $x^3 - 2 \in Q(x)$ is 6

7. Let p be a prime then $f(x) = x^p - 1 \in Q(x)$ has a splitting field $Q(\alpha)$ where $\alpha \neq 1$ and $\alpha^p = 1$. Also $[Q(\alpha) : Q] = p - 1$

8. Show that the splitting field of $f(x) = x^4 - 2 \in Q(x)$ *over* Q *is* $Q(2^{\frac{1}{4}}, i)$ and its degree of extension is 8

9. If the multiplicative group F^* of non zero elements of a field F is cyclic then F is Finite

10. The group of automorphisms of a field F with p^n elements is cyclic of order n and generated by ϕ where $\phi(x) = x^p$, $x \in F$

11. The group $G(\frac{Q(\alpha)}{Q})$ where $\alpha^5 = 1$ *and* $\alpha \neq 1$ is isomorphic to the cyclic group of order 4

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12. Let $E = Q(\sqrt[3]{2}, \omega)$ *where* $\omega^3 = 1, \omega \neq 1$ let σ_1 be the identity automorphism of E and Let σ_2 be an automorphism of E such that $\sigma_2(\omega) = \omega^2$ and $\sigma_2(\sqrt[3]{2}) = \omega(\sqrt[3]{2})$. If $G = \{\sigma_1, \sigma_2\}$ then $E_G = Q(\sqrt[3]{2}\omega^2)$
13. If $f(x) \in F(x)$ has r distinct roots in its splitting field E over F then the Galois group $G\left(\frac{E}{F}\right)$ *of* $f(x)$ is a subgroup of the symmetric group S_r.
14. The Galois group of $x^4 - 2 \in Q(x)$ is the octic group.
15. The Galois group of $x^4 + 1 \in Q(x)$ is Klein four group
16. $\phi_8(x)$ and $x^8 - 1$ have the same Galois group namely $\left(\frac{Z}{\langle 8 \rangle}\right)^* = \{1,3,5,7\}$, *the* Klein's four group.
17. If a field F contains a primitive n^{th} root of unity then the characteristic of F is Zero or a prime P that does not divide n
18. Show that the following polynomials are not solvable by radicals over Q
    (i) $x^7 - 10x^5 + 15x + 5$ (ii) $x^5 - 9x + 3$ (iii) $x^5 - 4x + 2$

19. It is impossible to construct a cube with a volume equal to twice the volume of a given cube by using ruler and compass only.

20. A regular n-gon is constructible if and only if $\phi(n)$ is a power of 2. (equivalently the angle $\frac{2\pi}{n}$ is Constructible.)

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DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. Mathematics

MM –202 **Semeste II**

Advanced Analysis

Paper II

Unit I

Algebra of sets- Borel sets- Outer measure- Measurable sets and Lebesgue measure- A non-measurable set- Measurable functions- Little word's three principles.

Unit II

The Rieman integral- The Lebesgue integral of a bounded function over a set of finite measure- The integral of a non-negative function- The general Lebesgue integral.

Unit III

Convergence in measure- Differentiation of a monotone functions- Functions of bounded variation.

Unit-IV

Differentiation of an integral- Absolute continuity- The L^p-spaces- The Minkowski and Holder's inequalities- Convergence and completeness.

Text Books:[1] Real Analysis (3rd Edition)(Chapters 3, 4, 5)

by

H. L. Royden

Pearson Education (Low Price Edition)

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DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. Mathematics

MM301 Semester III Complex Analysis ✓

Paper-I

UNIT-I

Regions in the Complex Plane -Functions of a Complex Variable - Mappings -Mappings by the Exponential Function- Limits - Limits Involving the Point at Infinity - Continuity - Derivatives - Cauchy–Riemann Equations -Sufficient Conditions for Differentiability - Analytic Functions - Harmonic Functions - Uniquely Determined Analytic Functions - Reflection Principle - The Exponential Function -The Logarithmic Function -Some Identities Involving Logarithms - Complex Exponents - Trigonometric Functions -Hyperbolic Functions

UNIT-II

Derivatives of Functions w(t) - Definite Integrals of Functions w(t) - Contours -Contour Integrals - Some Examples -Examples with Branch Cuts - Upper Bounds for Moduli of Contour Integrals –Anti derivatives -Cauchy–Goursat Theorem -Simply Connected Domains- Multiply Connected Domains -Cauchy Integral Formula -An Extension of the Cauchy Integral Formula -Liouville's Theorem and the Fundamental Theorem of Algebra -Maximum Modulus Principle

UNIT-III

Convergence of Sequences - Convergence of Series - Taylor Series -Laurent Series -Absolute and Uniform Convergence of Power Series- Continuity of Sums of Power Series - Integration and Differentiation of Power Series - Uniqueness of Series Representations-Isolated Singular Points - Residues - Cauchy's Residue Theorem - Residue at Infinity - The Three Types of Isolated Singular Points - Residues at Poles - Examples - Zeros of Analytic Functions -Zeros and Poles -Behavior of Functions Near Isolated Singular Points

UNIT-IV

Evaluation of Improper Integrals -Improper Integrals from Fourier Analysis - Jordan's Lemma - Indented Paths - - Definite Integrals Involving Sines and Cosines - Argument Principle - Rouche's Theorem -Linear Transformations -The Transformation $w = 1/z$ - Mappings by $1/z$ - Linear Fractional Transformations -An Implicit Form - Mappings of the Upper Half Plane

Text: James Ward Brown, Ruel V Churchill, Complex Variables with applications

Department of Mathematics
PALAMURU UNIVERSITY
M.Sc Mathematics

MM302 Semester III

Functional Analysis
Paper-II

Unit –I

NORMED LINEAR SPACES: Definitions and Elementary Properties, Subspace, Closed Subspace, Finite Dimensional Normed LinearSpaces and Subspaces, Quotient Spaces, Completion of Normed Spaces.

Unit-II

HILBERT SPACES: Inner Product Space, Hilbert Space, Cauchy-Bunyakovsky-Schwartz Inequality, Parallelogram Law, Orthogonality, Orthogonal Projection Theorem, Orthogonal Complements, Direct Sum, Complete Orthonormal System, Isomorphism between Separable HilbertSpaces.

Unit-III

LINEAR OPERATORS: Linear Operators in Normed Linear Spaces, Linear Functionals, The Space of Bounded Linear Operators, Uniform Boundedness Principle,Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector and Normed Linear Space, The General Form of Linear Functionals in Hilbert Spaces.

Unit-IV

FUNDAMENTAL THEOREMS FOR BANACH SPACES AND ADJOINT OPERATORS IN HILBERT SPACES: Closed Graph Theorem, Open Mapping Theorem, Bounded Inverse Theorem, Adjoint Operators, Self-Adjoint Operators, Quadratic Form, Unitary Operators, Projection Operators.

Text Book:

A First Course in Functional Analysis-Rabindranath Sen, Anthem Press An imprint of Wimbledon Publishing Company.

Reference:

1. Introductory Functional Analysis- E.Kreyzig- John Wilely and sons, New York.
2. Functional Analysis, by B.V. Limaye 2nd Edition.
3. Introduction to Topology and Modern Analysis- G.F.Simmons, Mc.Graw-Hill International Edition.

DEPARTMENT OF MATHEMATICS
PALAMURU UNIVERSITY

M.Sc. Mathematics

MM –303 A Semester -III

Discrete Mathematics

Paper-III (A)

UNIT- I

LATTICES: Partial Ordering – Lattices as Posets – some properties of Lattices – Lattices as Algebraic Systems – Sublattices, Direct products and Homomorphisms – some special Lattices – Complete, complemented and distributive lattices.
(Pages 183-192, 378-397 of [1])

UNIT- II

BOOLEAN ALGEBRA: Boolean Algebras as Lattices – Bolean Identities – the switching Algebra – sub algebra, Direct product and homomorphism – Join irreducible elements – Atoms (minterms) – Boolean forms and their equivalence – minterm Boolean forms – Sum of products canonical forms – values of Boolean expressions and Boolean functions – Minimization of Boolean functions – the Karnaugh map method.
(Pages 397 – 436 of [1])

UNIT- III

GRAPHS AND PLANAR GRAPHS : Directed and undirected graphs – Isomorphism of graphs – subgraph – complete graph – multigraphs and weighted graphs paths simple and elementary paths circuits connectedness – shortest paths in weighted graphs – Eulerian paths and circuits – Incoming degree and outgoing degree of a vertex - Hamiltonian paths and circuits – Planar graphs – Euler's formula for planar graphs.
(Pages 137-159, 168-186 of [2])

UNIT- IV

TREES AND CUT-SETS: Properties of trees – Equivalent definitions of trees - Rooted trees – Binary trees – path lengths in rooted trees – Prefix codes – Binary search trees – Spanning trees and Cut-sets – Minimum spanning trees
(Pages 187-213 of [2])

Text Books:-

[1] J P Tremblay and R. Manohar: Discrete Mathematical Structures with applications to Computer Science, McGraw Hill Book Company
[2] C L Liu : Elements of Discrete Mathematics, Tata McGraw Hill Publishing Company Ltd. New Delhi. (Second Edition).

DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. (Mathematics)

MM – 304A **Semester III**

Operations Research

Paper IV A

Unit I

Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method, Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions.Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

Unit II

Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.

Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

Unit III

Concept of Dynamic programming, Bellman's principle of optimality, characteristics of Dynamic programming problem, Backward and Forward recursive approach, Minimum path problem, Single Additive constraint and Multiplicatively separable return, Single Additive constraint and Additively separable return, Single Multiplicatively constraint and Additively separable return.

Unit-IV

Historical development of CPM/PERT Techniques - Basic steps - Network diagram representation - Rules for drawing networks - Forward pass and Backward pass computations - Determination of floats - Determination of critical path - Project evaluation and review techniques updating.

Departemnt of MATHEMATICS,PU
Proposed Choice Based Credit System (CBCS)
M.Sc Mathematics

Semester -IV

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 401	I	Advanced Complex Analysis	4	20	80	100	4
2. Core	MM 402	II	General Measure Theory	4	20	80	100	4
3.Elective	MM 403 A MM 403 B MM 403 C	III	Integral equations and Calculus of Varaiations Mechanics Finite Difference Method	4	20	80	100	4
4. Elective	MM 404 A MM 404 B MM 404 C	IV	Elementary Opearator Theory Prime Numbe Theory Advanced Opeartion Research	4 OR	20	80	100	4 OR
4. Elective	MM 404 D	IV	Project	6	....	....	150	6
5. Practical	MM 451	Practical	Advanced Complex Analysis	4	....	50	50	2
6. Practical	MM 452	Practical	General Measure Theory	4	....	50	50	2
7. Practical	MM 453 A MM 453 B MM 453 C	Practical	Integral equations and Calculus of Varaiations Mechanics Finite Difference Method	4	....	50	50	2
8. Practical	MM 454 A MM 454 B MM 454 C	Practical	Elementary Opearator Theory Prime Number Theory Advanced Opeartion Research	4	....	50	50	2
			Total :	**32**				**24**
9.Seminar			Seminar	2	.....	.....	25	1

30-11-2017 30/11/2017 30.11.17

DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. Mathematics

MM401 Semester IV

Advanced Complex Analysis

Paper-I

UNIT-I

Entire Functions: Jensen's formula -Functions of finite order- Infinite products Generalities -Example: the product formula for the sine function -Weierstrass infinite products - Hadamard's factorization theorem

UNIT-II

The Gamma and Zeta Functions: The gamma function –Analytic continuation-Further properties of Γ -The zeta function -Functional equation and analytic continuation

UNIT-III

The Zeta Function and Prime Number Theorem: Zeros of the zeta function - Estimates for $1/\zeta(s)$ - Reduction to the functions ψ and ψ_1 -Proof of the asymptotics for ψ_1 - Note on interchanging double sums

UNIT-IV

Conformal Mappings: Conformal equivalence and examples -The disc and upper half-plane - Further examples -The Dirichlet problem in a strip -The Schwarz lemma; automorphisms of the disc and upper half-plane-Automorphisms of the disc - Automorphisms of the upper half-plane

Text Book : Elias M Stein, Rami Shakarchi , *Complex Analysis*

References: Lars V Ahlfors, *Complex Analysis*

R P Boas, *Entire Functions*

Lars V Ahlfors, *Conformal Invariants*

Advanced Complex Analysis

Paper-I

MM451 Semester IV

Practical Questions

1

Prove that if $|z| < 1$, then

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\cdots = \prod_{k=0}^{\infty}(1+z^{2^k}) = \frac{1}{1-z}.$$

2

Find the Hadamard products for:

(a) $e^z - 1$;

(b) $\cos \pi z$.

3

Prove that for every z the product below converges, and

$$\cos(z/2)\cos(z/4)\cos(z/8)\cdots = \prod_{k=1}^{\infty}\cos(z/2^k) = \frac{\sin z}{z}.$$

4

Show that the equation $e^z - z = 0$ has infinitely many solutions in C.

5

Prove Wallis's product formula

$$\frac{\pi}{2} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdots\frac{2m\cdot 2m}{(2m-1)\cdot(2m+1)}\cdots$$

6

Prove that

$$\Gamma(s) = \lim_{n\to\infty}\frac{n^s n!}{s(s+1)\cdots(s+n)}$$

whenever $s = 0, -1, -2, \ldots$.

7

Show that Wallis's product formula can be written as

$$\sqrt{\frac{\pi}{2}} = \lim_{n\to\infty} \frac{2^{2n}(n!)^2}{(2n+1)!}(2n+1)^{1/2}.$$

8

Use the fact that $\Gamma(s)\Gamma(1-s) = \pi/\sin \pi s$ to prove that

$$|\Gamma(1/2+it)| = \sqrt{\frac{2\pi}{e^{\pi t}+e^{-\pi t}}}, \qquad \text{whenever } t \in \mathbb{R}.$$

9

The Beta function is defined for $\mathrm{Re}(\alpha) > 0$ and $\mathrm{Re}(\beta) > 0$ by

$$B(\alpha,\beta) = \int_0^1 (1-t)^{\alpha-1}t^{\beta-1}\,dt.$$

(a) Prove that $B(\alpha,\beta) = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.

(b) Show that $B(\alpha,\beta) = \displaystyle\int_0^\infty \frac{u^{\alpha-1}}{(1+u)^{\alpha+\beta}}\,du.$

10

Prove that for $\mathrm{Re}(s) > 1$,

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty \frac{x^{s-1}}{e^x-1}\,dx.$$

11

Prove as a consequence that one has

$$(\zeta(s))^2 = \sum_{n=1}^{\infty} \frac{d(n)}{n^s} \quad \text{and} \quad \zeta(s)\zeta(s-a) = \sum_{n=1}^{\infty} \frac{\sigma_a(n)}{n^s}$$

for $\mathrm{Re}(s) > 1$ and $\mathrm{Re}(s-a) > 1$, respectively. Here $d(n)$ equals the number of divisors of n, and $\sigma_a(n)$ is the sum of the a^{th} powers of divisors of n. In particular, one has $\sigma_0(n) = d(n)$.

12

Show that if $\{a_m\}$ and $\{b_k\}$ are two bounded sequences of complex numbers, then

$$\left(\sum_{m=1}^{\infty} \frac{a_m}{m^s}\right)\left(\sum_{k=1}^{\infty} \frac{b_k}{k^s}\right) = \sum_{n=1}^{\infty} \frac{c_n}{n^s} \quad \text{where } c_n = \sum_{mk=n} a_m b_k.$$

The above series converge absolutely when $\mathrm{Re}(s) > 1$.

13

Prove that for $\mathrm{Re}(s) > 1$

$$\frac{1}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s},$$

where $\mu(n)$ is the Möbius function defined by

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1, \\ (-1)^k & \text{if } n = p_1 \cdots p_k, \text{ and the } p_j \text{ are distinct primes,} \\ 0 & \text{otherwise.} \end{cases}$$

Note that $\mu(nm) = \mu(n)\mu(m)$ whenever n and m are relatively prime. [Hint: Use the Euler product formula for $\zeta(s)$.]

14

Show that

$$\sum_{k|n} \mu(k) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

15

Prove that the Dirichlet series

$$\sum_{n=1}^{\infty} \frac{a_n}{n^s}$$

converges for $\mathrm{Re}(s) > 0$ and defines a holomorphic function in this half-plane.

16

Does there exist a holomorphic surjection from the unit disc to C?

17

Prove that $f(z) = -\frac{1}{2}(z + 1/z)$ is a conformal map from the half-disc $\{z = x + iy : |z| < 1,\ y > 0\}$ to the upper half-plane.

18

Prove that the function u defined by

$$u(x, y) = \mathrm{Re}\left(\frac{i+z}{i-z}\right) \quad \text{and} \quad u(0, 1) = 0$$

is harmonic in the unit disc and vanishes on its boundary. Note that u is not bounded in D.

19

Show that if $f : D(0, R) \to \mathbb{C}$ is holomorphic, with $|f(z)| \leq M$ for some $M > 0$,

then

$$\left|\frac{f(z) - f(0)}{M^2 - \overline{f(0)}f(z)}\right| \leq \frac{|z|}{MR}.$$

20

Prove that if $f : D \to D$ is analytic and has two distinct fixed points, then f is the identity, that is, $f(z) = z$ for all $z \in D$.

DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. Mathematics

MM – 402 Semester IV

General Measure Theory

Paper- II

Unit I

Measure spaces- Measurable functions- Integration- General Convergence theorem.

Unit II

Signed measures- The Radon- Nikodym theorem.

Unit III

Outer measure and measurability- The Extension theorem- The Product measure.

Unit-IV

Inner measure- Extension by sets of measure zero- Caratheodory outer measure

Text Books:

[1] Real Analysis (Chapters 11, 12)

By H.L. Royden , Pearson Ediation.

M.Sc.(Mathematics)

General Measure Theory

Paper II

MM 452 Semester

IVPractical Questions

1. Let be a non negative Lebesgue measurable function on $\mathbb{R}$. For each Lebesgue measurable set of $\mathbb{R}$ define $( ) = \int$ the Lebesgue integral of over Show that is a measure onthe -algebra of Lebesgue measurable subsets of $\mathbb{R}$.

2. Let be a -algebra of subsets of a set and : $\rightarrow [0, \infty)$ be finitely additive set function. Prove that is a measure if and onlyif whenever { } is an ascending sequence in then

$\bigcup$) = lim $_{\rightarrow}$ ( ).

3. Let (, ,) be a measure space and Δ denote the symmetric difference of two subsets and of That is Δ = [−] ∪ [−]. Show that

i. If and are measurable and (Δ) = 0 then ( ) = ( )

ii. show that if is complete, ∈ and − ∈ .Then ∈ if (Δ) = 0

4. Let (, ,) be a measure space and ∈ .Define to be the collection of sets in that are subsets of and the restriction of to . Show that (, ,) is a measure space.

5. Suppose (, ,) is not complete. Let be a subset of a set of measure zero that does not belong to .Let = 0on and = . Show that = a.e on which is measurable and is not.

6. Let : → be a function that is Lebesgue integrable over . For a Lebesgue measurable set define ( ) = $\int$. Prove that is a signed measure on the Lebesgue measurable space (,). Find a Hahn-decomposition of with respect to this signed measure.

7. Let be a measure and and are mutually singular measures on a measurable space (,) for which = − . Show that = 0. Use this to establish uniqueness assertion of th Jordan decomposition theorem.

8. Show that if is any measurable set then

i. − ( ) ≤ ( ) ≤ ( )

ii. | ( )| ≤ | |( )

9. Show that if and are two finite signed measures then so is where and ar real numbers. Show that | | = | || | and | + | ≤ | | + | |

10. In the question 6 if is a Lebesgue measurable set such that 0 < ( ) < ∞. Find a positive set contained in such that ( ) > 0. Also find a Jordan decomposition of .

11.Let be a collection of subsets of a set and : $\rightarrow [0,\infty]$ be a set of function. Define $^*(\emptyset) = 0$ and for $\subseteq$, $\neq \emptyset$ Define $^*( ) = \inf \sum$ ( ) where the infimum is taken over all countable collections { } of sets in that cover . Prove that the set function *: ( ) $\rightarrow [0,\infty]$ is an outer measure called the outer measure induced by .

12. Let *: ( ) $\rightarrow [0,\infty]$ be an outer measure. Let $\subseteq$,{ } be a disjoint countable collection of measurable sets and $= \cup$. Show that $^*(\cap) = \sum$ $^*(\cap)$.

13. Show that any measure that is induced by an outer measure is complete.

14. Let be a set, $= \{\emptyset,$ } and define $(\emptyset) = 0$ and $( ) = 1$. Determine the outer measure * induced by the set function : $\rightarrow [0,\infty]$ and the -algebra of measurable sets.

15. On the collection *S* of all subsets of define the set function : $\rightarrow$ by setting ( ) to be the number of integers in . Determine the outer measure * induced by and the -algebra of measurable sets.

16. Let be a -algebra on and a collection of subsets of which is closed under countable unions and which has the property that each subset of a set in is in . Show that the collection

$= \{ \ : \ = \ \Delta \ , \ \in \ ,$ is a }algebra.

17. i. If $( ) < \infty$ prove that $_*( ) = ( ) - {}^*( )$ and

ii. if is a -algebra then prove that

$^*( ) = \inf\{ ( ): \ \subset \ , \ \in$ and }

$_*( ) = \sup\{ ( ): \ \subset \ , \ \in \ \}$

18. Suppose is a measure on an algebra of subsets of and is any subset of . If is the algebra generated by and and if $\bar{}$ and $\underline{}$ are extensions of to such that $\bar{}( ) = {}^*( )$ and $\underline{}( ) = {}_*( )$. Prove that $\bar{}$ and $\underline{}$ are measures on .

19. Suppose $= \{ \ = (\ \cap \ ) \cup (\ \cap \ ): \ , \ \in \ \}$ where and are as in problem 18. Prove that is an algebra of subsets of containing and .

20. Suppose (,) is a metric space and * be an outer measure on with the property that $^*(\ \cup \ ) = {}^*( ) + {}^*( )$ whenever $(\ , \ ) > 0$. Prove that every closed set is measurable with respect to *.

DEPARTMENT OF MATHEMATICS

PALAMURU UNIVERSITY

M.Sc. (Mathematics)

MM – 403A | Paper IIIA | Semester IV

Integral Equations & Calculus of Variations

Integral Equations:

Unit I

Volterra Integral Equations: Basic concepts - Relationship between Linear differential equations and Volterra Integral equations - Resolvent Kernel of Volterra Integral equation. Differentiation of some resolvent kernels - Solution of Integral equation by Resolvent Kernel - The method of successive approximations - Convolution type equations - Solution of Integro-differential equations with the aid of the Laplace Transformation – Volterra integral equation of the first kind - Euler integrals - Abel's problem - Abel's integral equation and its generalizations.

Unit II

Fredholm Integral Equations: Fredholm integral equations of the second kind – Fundamentals – The Method of Fredholm Determinants - Iterated Kernels constructing the Resolvent Kernel with the aid of Iterated Kernels - Integral equations with Degenerated Kernels. Hammerstein type equation - Characteristic numbers and Eigen functions and its properties.

Green's function: Construction of Green's function for ordinary differential equations - Special case of Green's function - Using Green's function in the solution of boundary value problem.

Calculus of Variations:

Unit III

The Method of Variations in Problems with fixed Boundaries:

Definitions of Functionals – Variation and Its properties - Euler's' equation - Fundamental Lemma of Calculus of Variation-The problem of minimum surface of revolution - Minimum Energy Problem Brachistochrone Problem - Variational problems involving Several functions - Functional dependent on higher order derivatives - Euler Poisson equation.

Unit IV

Functional dependent on the Functions of several independent variables - Euler's equations in two dependent variables - Variational problems in parametric form - Application of Calculus of Variation - Hamilton's principle - Lagrange's Equation, Hamilton's equations.

Text Books:

[1] M. KRASNOV, A. KISELEV, G. MAKARENKO, Problems and Exercises in Integral Equations (1971)

[2] S. Swarup, Integral Equations, (2008)

[3] L.ELSGOLTS, Differential Equation and Calculus of Variations, MIR Publishers, MOSCOW

M.Sc.(Mathematics)

Integral Equations & Calculus of Variations

MM453A Paper IIIA Semester IV

Practical Questions

1. From an Integral equation corresponding to the differential equation
$y''' + xy'' + (x^2 - x)y = xe^x + 1$; $y(0) = y'(0) = 1$; $y''(0) = 1$
2. Convert the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; with initial conditions $y(0) = y'(0) = 1,\ y''(0) = 0$; into Volterra's Integral Equations.
3. Solve the Integral Equations
$$\varphi''(x) + \varphi(x) + \int Sinh(x-t)\varphi(t)dt + \int_0^x Cosh(x-t)\varphi'(t)dt = Coshx\ ;\ \varphi(0) = \varphi'(0) = 0.$$
4. Solve the Integral Equations $\int_0^x \frac{\varphi(t)dt}{(x-t)^\alpha} = x^n\ ;\ 0 < \alpha < 1;$
5. With the aid of Resolvent Kernel, find the solution of the Integral equation
$$\varphi(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\varphi(t)dt$$
6. Solve the Integral Equations $\phi(x) - \lambda\int_0^1 \arccos t.\phi(t)dt = \frac{1}{\sqrt{1-x^2}}$
7. Find the Characteristic numbers and Eigen function of the Integral Equations
$$\phi(x) - \lambda\int_0^1 \left(45x^2 \log t - 9t^2 \log x\right)\phi(t)dt = 0$$
8. **Applications of Green's function :** Construct Green's function for the homogeneous boundary value problem $y^{iv}(x) = 0;\ y(0) = y'(0) = 0;\ y(1) = y'(1) = 0$.
9. **Applications of Green's function :** Solve the Boundary Value problem $y^{iv}(x) = 1;$
$y(0) = y'(0) = y''(1) = y'''(0) = 0$.
10. **Applications of Green's function :** Solve the Boundary Value problem $y'' + y = x^2$;
$y(0) = y(\pi/2) = 0$.
11. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y}dx$
12. Test for an extremum the functional $v[y(x)] = \int_0^1 \left(xy + y^2 - 2y^2y'\right)dx\ ;\ y(0) = 1;\ y(1)) = 2.$

13. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1}(16y^2 - y''^2 + x^2)dx$

14. Determine the extremals of the functional $v[y(x)] = \int_{-l}^{l}(\frac{\mu}{2}y''^2 + \rho y)dx$ that satisfies the boundary conditions $y(-l) = y'(-l) = y(l) = y'(l) = 0$

15. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1}(2yz - 2y^2 + y'^2 - z'^2)dx$

16. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1}\left[y^2 + (y')^2 + \frac{2y}{Coshx}\right]dx$

17. Write the ***Ostrgradsky*** equation for the functional $v[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right]dxdy$

18. **Applications of Hamilton's and Lagrange's equations:** Derive the equation of a vibrations of a Rectilinear Bar.

19. **Applications of Hamilton's and Lagrange's equations:** A particle of mass **m** is moving vertically under the action of gravity and a resistance force numerically equal to **k** times the displacement **x** from an equilibrium position. Obtain the Hamilton's and Euler's equation.

20. Use Hamilton's principle to find the equations for the small vibrations of a flexible stretching string of length ***l*** and tension **T** fixed at end points.

Chairperson
BoS in Mathematics
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M.Sc.(Mathematics)

Integral Equations & Calculus of Variations

MM453A | Paper IIIA | Semester IV

Practical Questions

1. From an Integral equation corresponding to the differential equation
$y''' + xy'' + (x^2 - x)y = xe^x + 1$; $y(0) = y'(0) = 1$; $y''(0) = 1$
2. Convert the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; with initial conditions $y(0) = y'(0) = 1$, $y''(0) = 0$; into Volterra's Integral Equations.
3. Solve the Integral Equations
$$\varphi''(x) + \varphi(x) + \int Sinh(x-t)\varphi(t)dt + \int_0^x Cosh(x-t)\varphi'(t)dt = Coshx \ ; \ \varphi(0) = \varphi'(0) = 0.$$
4. Solve the Integral Equations $\int_0^x \frac{\varphi(t)dt}{(x-t)^\alpha} = x^n$; $0 < \alpha < 1$;
5. With the aid of Resolvent Kernel, find the solution of the Integral equation
$$\varphi(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\varphi(t)dt$$
6. Solve the Integral Equations $\phi(x) - \lambda\int_0^1 \arccos t.\phi(t)dt = \frac{1}{\sqrt{1-x^2}}$
7. Find the Characteristic numbers and Eigen function of the Integral Equations
$$\phi(x) - \lambda\int_0^1 \left(45x^2 \log t - 9t^2 \log x\right)\phi(t)dt = 0$$
8. **Applications of Green's function :** Construct Green's function for the homogeneous boundary value problem $y^{iv}(x) = 0$; $y(0) = y'(0) = 0$; $y(1) = y'(1) = 0$.
9. **Applications of Green's function :** Solve the Boundary Value problem $y^{iv}(x) = 1$; $y(0) = y'(0) = y''(1) = y'''(0) = 0$.
10. **Applications of Green's function :** Solve the Boundary Value problem $y'' + y = x^2$; $y(0) = y(\pi/2) = 0$.
11. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y}dx$
12. Test for an extremum the functional $v[y(x)] = \int_0^1 \left(xy + y^2 - 2y^2y'\right)dx$; $y(0) = 1$; $y(1)) = 2$.